HW. # 10

Homework problems are taken from several textbooks. The problems are color coded to indicate level of difficulty. The color green indicates an elementary problem, which you should be able to solve effortlessly. Yellow means that the problem is somewhat harder. Red indicates that the problem is hard. You should attempt the hard problems especially.

Obtain formulas for $\frac{du}{dt}$.

1. $u = x^4 - y^4$, x = cos(t), y = $\sin(t^2)$

2. $u = e^{x+4y-z}$, x = $\ln t$, y = $\ln\left(\frac{t+1}{t}\right)$, z = $\frac{1}{t}$

3. $u = \tan^{-1}\left(\frac{y}{x}\right)$, x = $\sqrt{t}$, y = $t^{\frac{3}{2}}$

4. $u = f(g(t))$, g(t) = (sin(t), cos(t), t)

5. $u = \frac{x+y}{x-y}$, x = f(t), y = g(t)

Obtain formulas for $\frac{\partial u}{\partial t}$ and $\frac{\partial u}{\partial s}$.

6. $u = x^2 y^3 + x - 3y$; x = $t^2 - s$, y = $t - s^2$

7. $u = x + y - z$; x = $3s + 2t$, y = $-s + t$, z = $s + t$

8. $u = \frac{x+1}{x^2 + y^2}$; x = $s^2 + t$, y = $s - t^2$

9. $u = \ln(xyz)$; x = $s + t$, y = $s - 3t$, z = $\frac{1}{s-t}$

10. $u = g(s^2 - t^2, 2st)$

11. $u = \dfrac{s}{x(t)^2 + y(t)^2 + z(t)^2}$

12. $u = tf$ (s + t, 2s + 3t) – sf (2s + 3t, s + t)

13. $u = e^t g\,(e^2 + e^{-s}$, ln(t), ln($e^{t+s} + 1$))

Use the chain rule to find the derivative of $g \circ f$ at the indicated point **a**.

14. $g(x, y) = (x^2 y^3, 3x - y^2)$, $f(x, y) = (-y, x)$, **a** = (3, 2)

15. $g(x, y) = (x^2 y^3, 3x - y^2)$, $f(x, y, z) = \left(xz, \dfrac{y}{xz}\right)$, **a** = (3, 1, -1)

16. $g(x, y, z) = (y^2, z^2, x^2)$, $f(x, y, z) = (\sin x, \cos y, \sin(x + y + z))$ **a** = $\left(\dfrac{\pi}{4}, \dfrac{-\pi}{4}, \dfrac{\pi}{6}\right)$

17. $g(x_1, x_2) = (x_1^3, x_2)$, $f(x_1, x_2, x_3) = (4x_1 + x_2 + x_3^2, x_1 x_3)$, **a** = (0, 1, 0)

18. Suppose that the temperature in space is given by T(x , y , z) = $\dfrac{1}{x^2 + y^2 + z^2}$ and let **x** = $(3t^2 - t, t^2, t^3)$ be a parameterization for a path. What is the rate of change in temperature along the path when t = 1?

19. Suppose that an ideal gas (see exercise 9 of HW#9) occupies a container with volume 900 cm^3 at temperature 400 K. If the volume is increasing at 10 cm^3/min and the temperature is increasing at 15 K/min, at what rate is the pressure in the container changing?

20. Economists attempt to measure how useful or satisfying people find goods or services with **utility functions**. Suppose that the utility a person derives from consuming x ounces of beer per week and watching y minutes of videotaped movies per week is

$$u(x, y) = 1 - e^{-0.0003x^2 - 0.00000009y^2}.$$

Further suppose that she currently drinks 70 ounces of beer per week and watches 300 minutes of movies per week. If she is increasing her consumption of beer by 4 ounces per week and cutting back on her movie watching by 10 minutes per week, is the utility she derives from these activities increasing or decreasing? At what rate?

Find the indicated partial derivatives, assuming that the given equation implicitly defines the appropriate differentiable function.

21. $x^3y^2z + xy - z^3 = 0$; $\dfrac{\partial z}{\partial x}, \dfrac{\partial z}{\partial y}$

22. $x\sin\dfrac{y}{z} + z\cos\dfrac{x}{y} = y$; $\dfrac{\partial z}{\partial x}, \dfrac{\partial z}{\partial y}$

23. Show that if f(x, y, z) = 0 defines each variable implicitly as a differentiable function of the other variables, then

$$\frac{\partial z}{\partial x}\frac{\partial x}{\partial y}\frac{\partial y}{\partial z} = -1$$

24. Find $\left.\dfrac{\partial z}{\partial x}\right|_S$ and $\left.\dfrac{\partial z}{\partial y}\right|_S$, where S is the surface $x^4 + y^4 + z^4 = 1$